

【SAP2000与结构力学】系列课程

SAP2000 梁结构分析

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北京筑信达工程咨询有限公司

2024年5月22日

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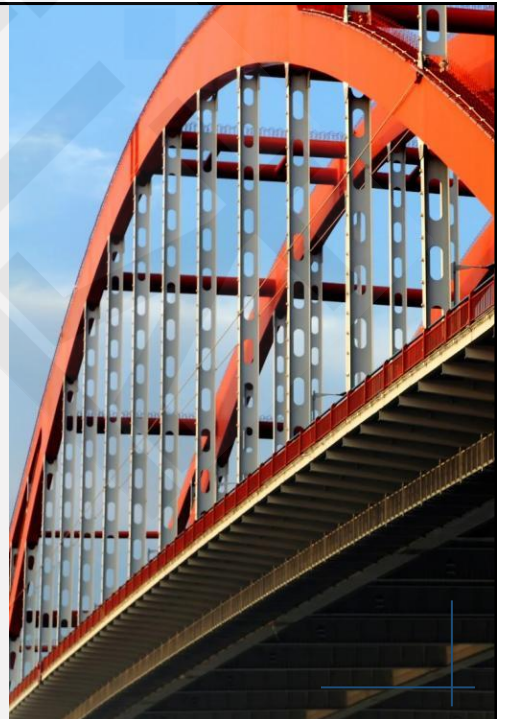
简支梁/连续梁/变截面梁

梁的横向剪切变形

线荷载与温度荷载

弯矩图和挠曲线

弹性支座



1

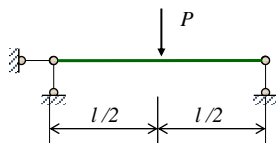
简支梁/连续梁/变截面梁



讯信达

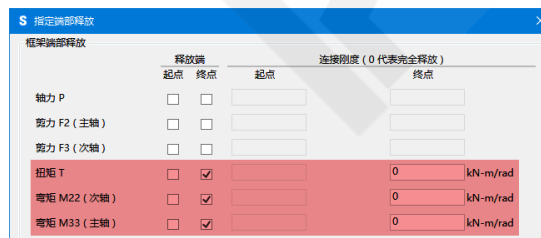
简支梁和连续梁

❖ 刚性支座



❖ 铰接节点

- 默认：刚接节点
- **端部释放**/连接单元/节点约束
- 避免节点的转动约束不足

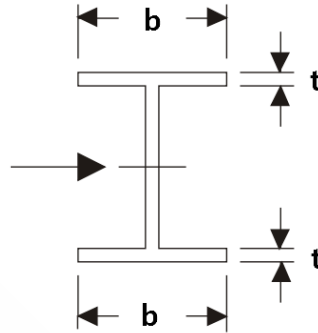
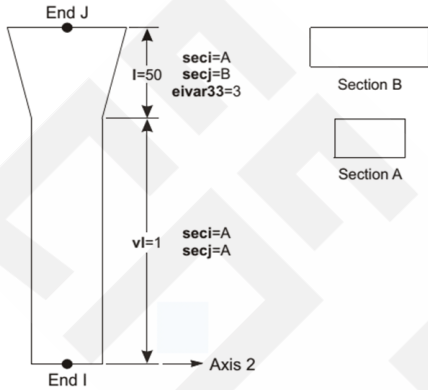


讯信达

变截面梁

❖ 抗弯刚度

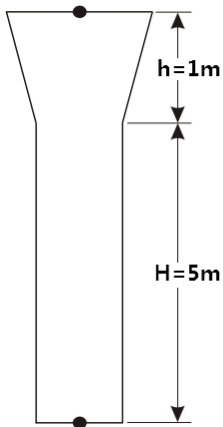
- 线性插值，二次插值，三次插值



筑信达

变截面梁

❖ 可变长度与绝对长度



Start Section	End Section	Length	Length Type	E33 Variation	E22 Variation
C400x400	C400x400	5	Absolute	Parabolic	Linear
C400x400	C400x400	5	Absolute	Parabolic	Linear
C400x400	C800x400	1	Absolute	Parabolic	Linear

Add Insert Modify Delete

筑信达

变截面梁

❖ 可变长度与绝对长度

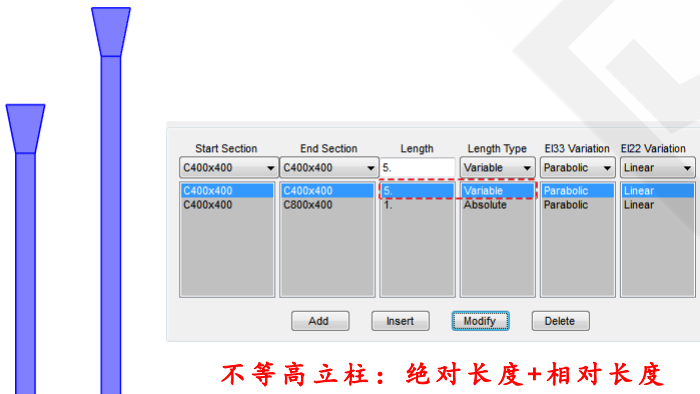


不等高立柱：绝对长度

筑信达

变截面梁

❖ 可变长度与绝对长度

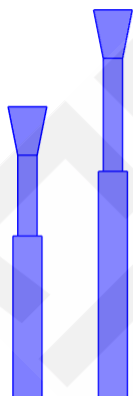


不等高立柱：绝对长度+相对长度

筑信达

变截面梁

❖ 可变长度与绝对长度



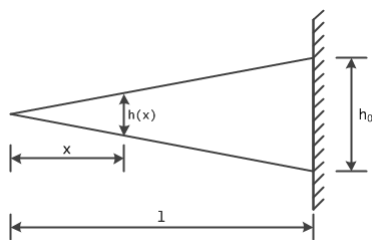
Start Section	End Section	Length	Length Type	EI3 Variation	EI22 Variation
C400x400	C800x400	1.	Absolute	Parabolic	Linear
C600x400	C600x400	2.	Variable	Parabolic	Linear
C400x400	C400x400	1.	Variable	Parabolic	Linear
C400x400	C800x400	1.	Absolute	Parabolic	Linear

不等高立柱2：绝对长度+相对长度

筑信达

变截面梁

❖ 楔形悬臂梁



模型信息

$$l = 6m, E = 30GPa, \rho = 2550kg/m^3, h(x) = \frac{h_0 x}{l}, h_0 = 1m$$

自振频率

$$\omega = \frac{1.534h_0}{l^2} \sqrt{\frac{E}{\rho}} = \frac{1.534 \times 1.0}{6^2} \sqrt{\frac{30 \times 10^9}{2550}} = 146.15$$

$$f = \frac{\omega}{2\pi} = \frac{146.15}{2 \times 3.1416} = 23.26Hz$$

	单元数量	单元尺寸 (m)	自振频率 (Hz)	误差 (%)
解析解	--	--	23.26	0.00
数值解一	1	6.0	3.39	-85.43
数值解二	5	1.2	16.26	-30.09
数值解三	20	0.3	23.15	-0.47
数值解四	50	0.12	23.25	-0.04

筑信达

2

梁的横向剪切变形



筑信达

梁的位移计算方法

❖ 材料力学

- 等直梁的挠曲线近似微分方程

$$EI\omega' = M(x) \longrightarrow EI\omega = -\int\int M(x)dx dx + C_1x + C_2$$

❖ 结构力学

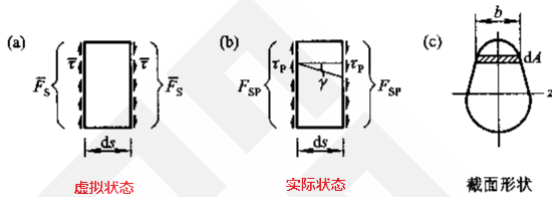
- 变形图虚功原理 → **单位荷载法** → 图乘法

$$\Delta_{XP} = \sum \int \bar{M}d\varphi_p + \sum \int \bar{F}_N du_p + \sum \int \bar{F}_S \gamma_p ds$$

筑信达

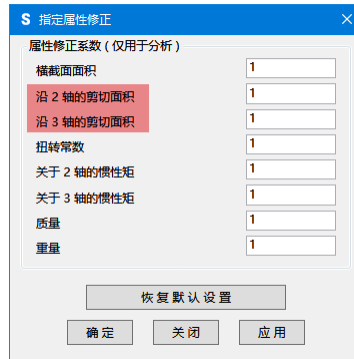
梁的横向剪切变形

$$\Delta_{FP} = \sum \int \bar{M} d\varphi_P + \sum \int \bar{F}_N du_P + \sum \int \bar{F}_S \gamma_P ds$$



$$\bar{F}_S \gamma_P ds = ds \int_A \frac{\bar{F}_S F_{SP} S^2 dA}{GI^2 b^2} = \frac{\bar{F}_S F_{SP} ds}{GA} \cdot \frac{A}{I^2} \int_A \frac{S^2}{b^2} dA = \frac{k \bar{F}_S F_{SP} ds}{GA}$$

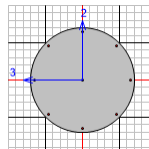
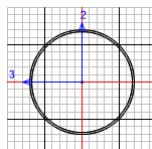
$$k = \frac{A}{I^2} \int_A \frac{S^2}{b^2} dA$$



筑信达

梁的横向剪切变形

组合截面的悬臂梁



$$EI = E_s I_s + E_c I_c = 2.06 \times 10^8 \times 0.05616 + 3 \times 10^7 \times 0.4042 = 2.3696 \times 10^7 \text{ kNm}^2$$

($\phi 150 \times 28$ 钢管复合)

$$\Delta = \frac{F_P L^3}{3EI} = \frac{1 \times 1^3}{3 \times 2.3696 \times 10^7} = 1.4067 \times 10^{-8} \text{ m}$$

Pt Obj: 2
Pt Elm: 2
U1 = 0
U2 = 0
U3 = 4.712E-08
R1 = 0
R2 = -2.11E-08
R3 = 0

筑信达

3

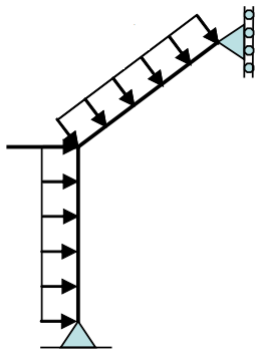
线荷载与温度荷载



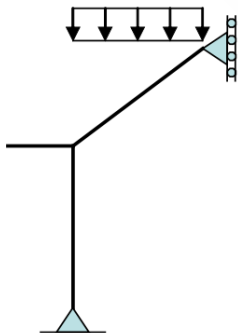
筑信达

线荷载

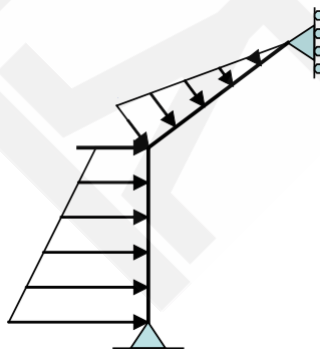
❖ 荷载大小与作用方向



坐标轴方向



投影方向

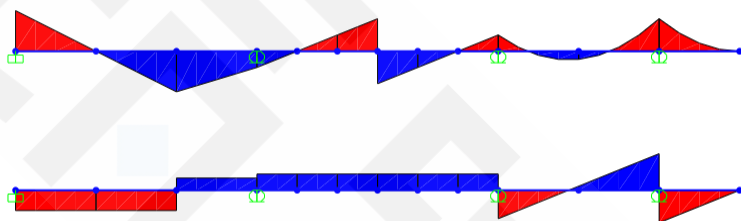


三角形分布和梯形分布

筑信达

线荷载

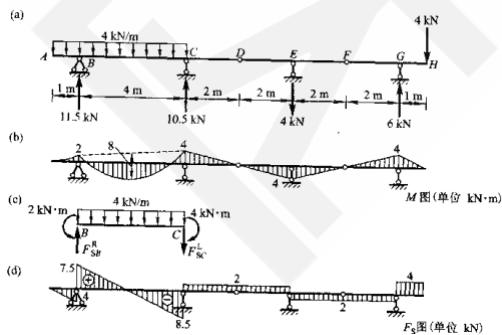
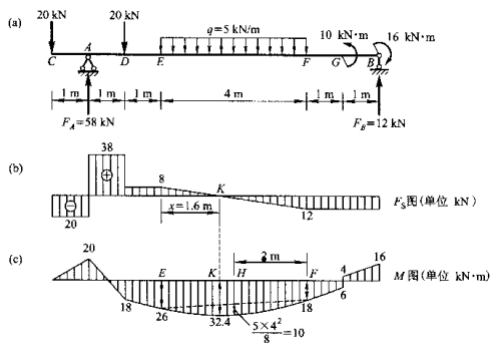
多跨静定梁



筑信达

线荷载

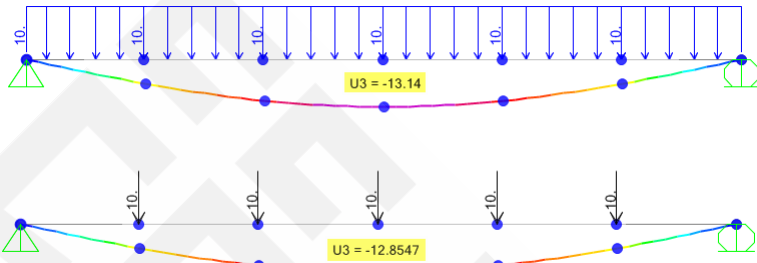
多跨静定梁



筑信达

线荷载

均布线荷载的等效集中荷载

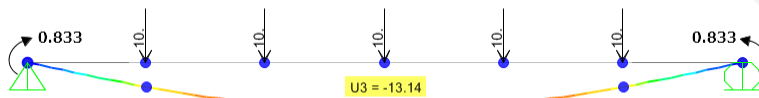


筑信达

线荷载

均布线荷载的等效集中荷载

梁的简图	弯矩		剪力	
	M_{AB}	M_{BA}	F_{SAB}	F_{SBA}
	$\frac{ql^2}{12}$	$\frac{ql^2}{12}$	$\frac{ql}{2}$	$-\frac{ql}{2}$

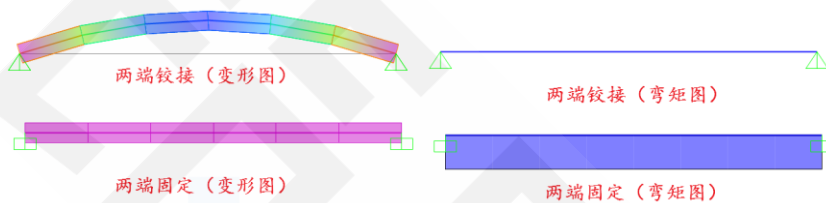


筑信达

温度荷载

❖ 温度荷载与温度应力

- 材料温差 \rightarrow 应变/变形 \rightarrow 应力/内力
- 静定结构：自由变形，内力/应力 **为零**
- 超静定结构：约束变形/无变形，内力/应力 **非零**

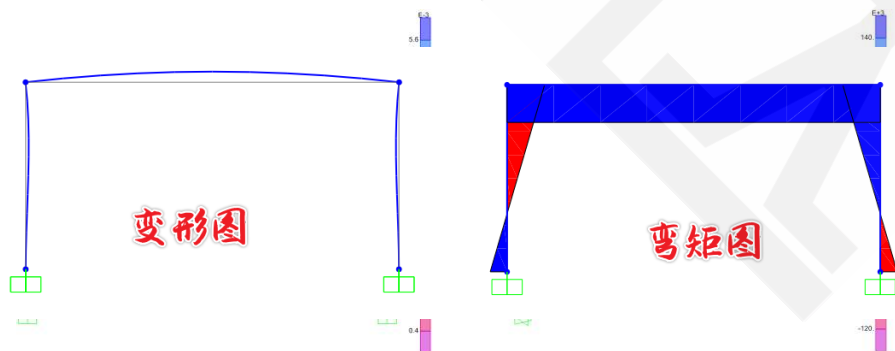


水平梁的顶面温度高于底面温度

筑信达

温度荷载

❖ 门式刚架



水平梁的顶面温度高于底面温度

筑信达

4

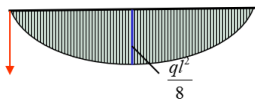
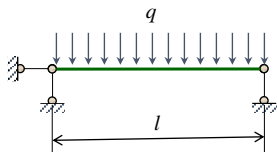
弯矩图和挠曲线



筑信达

测站

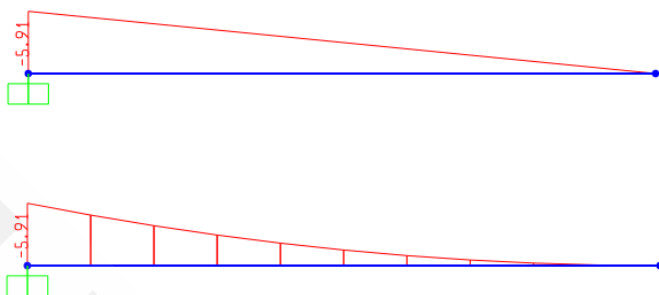
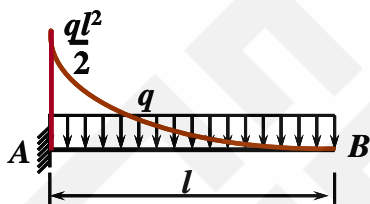
❖ 线性插值



筑信达

测站

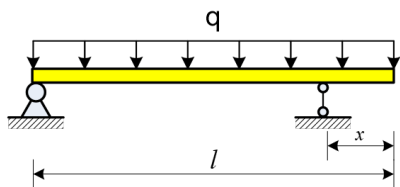
❖ 线性插值



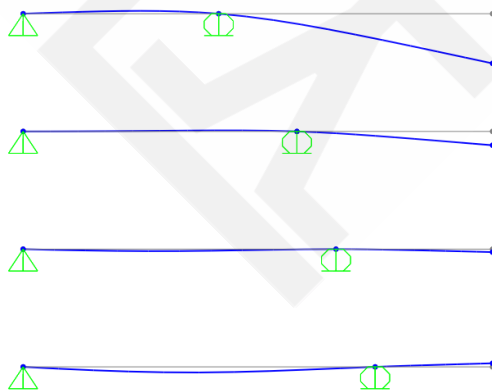
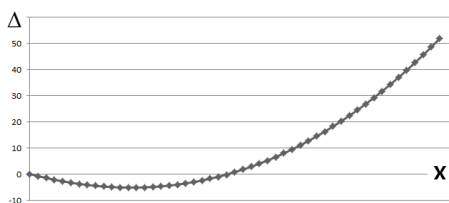
筑信达

节点位移

❖ 外伸梁的自由端挠度



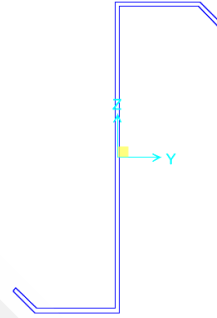
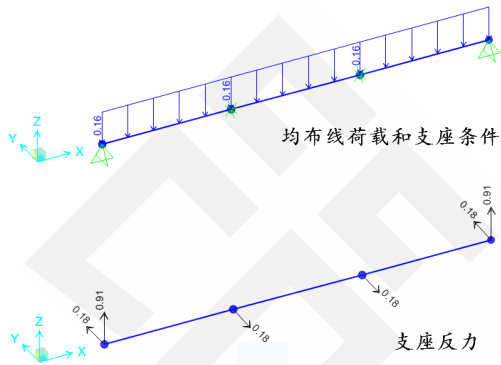
$$\Delta = \frac{1}{24EI} [4x^3(l-x) + 3x^4 - x(l-x)^3]$$



筑信达

挠曲线

❖ 非对称弯曲（斜弯曲）

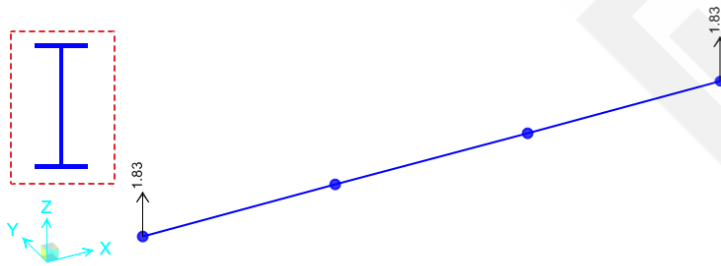


几何属性	
截面名称	Z-shape
几何属性	
横截面面积	1.066E-03
关于 3 轴的惯性矩	7.185E-06
关于 2 轴的惯性矩	9.199E-07
关于 2-3 轴的惯性积	-1.910E-06
沿 2 轴的剪切面积	5.220E-04
沿 3 轴的剪切面积	1.648E-04
扭转常数	3.197E-09

筑信达

挠曲线

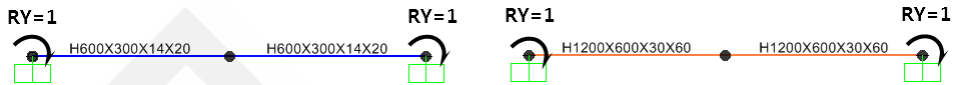
❖ 对称弯曲



筑信达

挠曲线

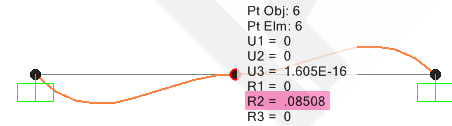
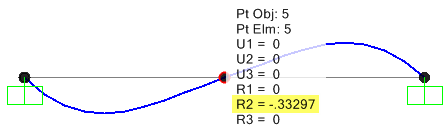
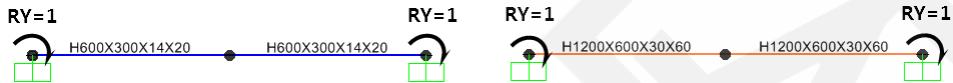
❖ 两端固定的等直梁



梁的简图	弯矩		剪力	
	M_{AB}	M_{BA}	F_{SAB}	F_{SBA}
	$4i$	$2i$	$-\frac{6i}{l}$	$-\frac{6i}{l}$
	$(i = \frac{EI}{l}, \text{下同})$			

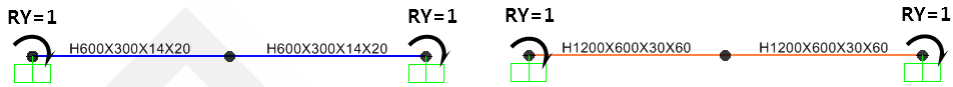
挠曲线

❖ 两端固定的等直梁



挠曲线

❖ 两端固定的等直梁



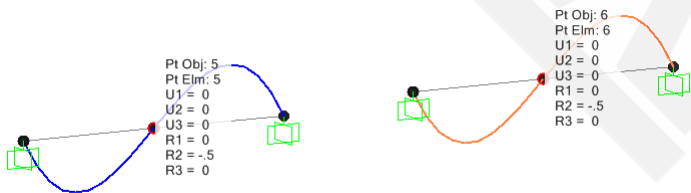
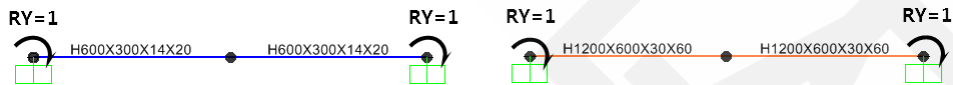
$$EI\omega'' = M(x) \longrightarrow EI\omega = -\int\int M(x)dx dx + C_1x + C_2$$

梁的简图	弯矩		剪力	
	M_{AB}	M_{BA}	F_{SAB}	F_{SBA}
	$4i$ $(i = \frac{EI}{l}, \text{下同})$	$2i$	$-\frac{6i}{l}$	$-\frac{6i}{l}$

筑信达

挠曲线

❖ 两端固定的等直梁



忽略剪切变形

筑信达

5

弹性支座



筑信达

弹性支座

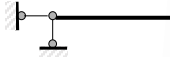
❖ 支座类型

• 刚性支座：零位移，零转角

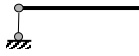
• 固定支座



• 固定铰支座



• 可动铰支座



• 定向支座



• 柔性支座：非零的位移或转角

• 线性（弹性）支座

• 非线性支座

6-22 图示梁 EI = 常数, B 处有一弹性支座, 弹簧的刚度系数 (产生单位位移所需的力) 为 k (注: 弹簧的柔度系数 f 即单位力作用下的位移, 是与刚度系数互为倒数的, 即 $f = \frac{1}{k}$)。试求 C 点竖向位移。已知 $k = \frac{EI}{a^3}$ 。



筑信达

谢谢

“The idea that an expert-system computer program, with artificial intelligence, will replace a **creative human is an insult to all structural engineers.”**



加州大学伯克利分校终身名誉教授 Edward L. Wilson (著名的结构分析设计软件 SAP 的创始人) 在《Three Dimensional Static and Dynamic Analysis Of Structures》(<http://www.edwilson.org/Book/book.htm#Personal>) 一书中提到:

"Don't use a structural analysis program unless you fully understand the theory and approximations used within the program"

"Don't create a computer model until the loading, material properties and boundary conditions are clearly defined"